Copernicus FICE 2025

Training on
In situ Ocean Colour Above-Water Radiometry towards Satellite Validation

GUM general metrological framework

Agnieszka Bialek
National Physical Laboratory
agnieszka.bialek@npl.co.uk

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Outline

- Methodology and resources
- Basic uncertainty concepts
- Absolute calibration measurement equation
- Above water radiometry measurement equation

Methodology and resources

Methodology and resources

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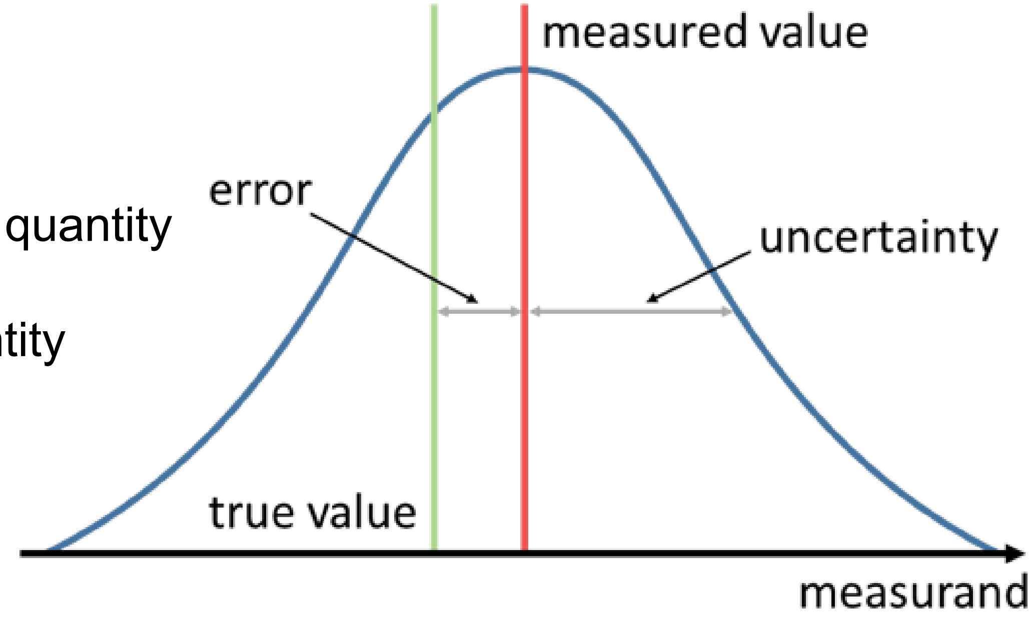
- The International Vocabulary of Metrology (VIM)

"the measured quantity value minus a reference quantity value."



"a non-negative parameter characterizing the dispersion of the quantity values being attributed to a measurand, based on the information used."

Bureau International des Poids et Mesures – the intergovernmental organization through which Member States act together on matters related to measurement science and measurement standards.

http://www.bipm.org/en/publications/guides/

Contents

Options

[VIM3] 2.26 measurement uncertainty

uncertainty of measurement, uncertainty

non-negative parameter characterizing the dispersion of the quantity values being attributed to a measurand, based on the information used

Notes

NOTE 1 Measurement uncertainty includes components arising from systematic effects, such as components associated with corrections and the assigned quantity values of measurement standards, as well as the definitional uncertainty. Sometimes estimated systematic effects are not corrected for but, instead, associated measurement uncertainty components are incorporated.

NOTE 2 The parameter may be, for example, a standard deviation called standard measurement uncertainty (or a specified multiple of it), or the half-width of an interval, having a stated coverage probability.

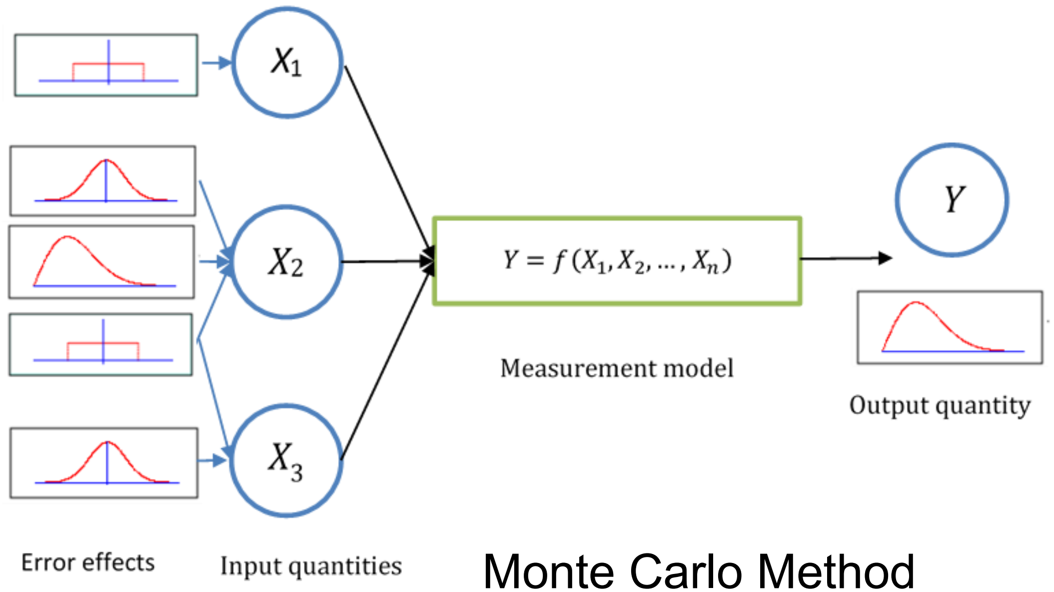
NOTE 3 Measurement uncertainty comprises, in general, many components. Some of these may be evaluated by Type A evaluation of measurement uncertainty from the statistical distribution of the quantity values from series of measurements and can be characterized by standard deviations. The other components, which may be evaluated by Type B evaluation of measurement uncertainty, can also be characterized by standard deviations, evaluated from probability density functions based on experience or other information.

NOTE 4 In general, for a given set of information, it is understood that the measurement uncertainty is associated with a stated quantity value attributed to the measurand. A modification of this value results in a modification of the associated uncertainty.

https://jcgm.bipm.org/vim/en/2.26.html

Methodology and resources

- the Guide to the expression of Uncertainty in Measurement (GUM) and its supplements



Monte Carlo Method

$$u^2(y) = \sum_{i=1}^{N} c_i^2 u^2(x_i) + 2 \sum_{i=1}^{N-1} \sum_{j=i}^{N} c_i c_j u(x_i, x_j),$$

The Law of Propagation of Uncertainties

http://www.bipm.org/en/publications/guides/

Bureau International des Poids et Mesures

– the intergovernmental organization through which Member States act together on matters related to measurement science and measurement standards.

Monte Carlo method

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"In mathematics, as in physics, so much depends on chance, on a propitious moment."

Stanislaw Ulam

Source: Adventures of a Mathematician , Third Edition (1991)
Chapter 5, p. 95,

QA4EO Home https://qa4eo.org/

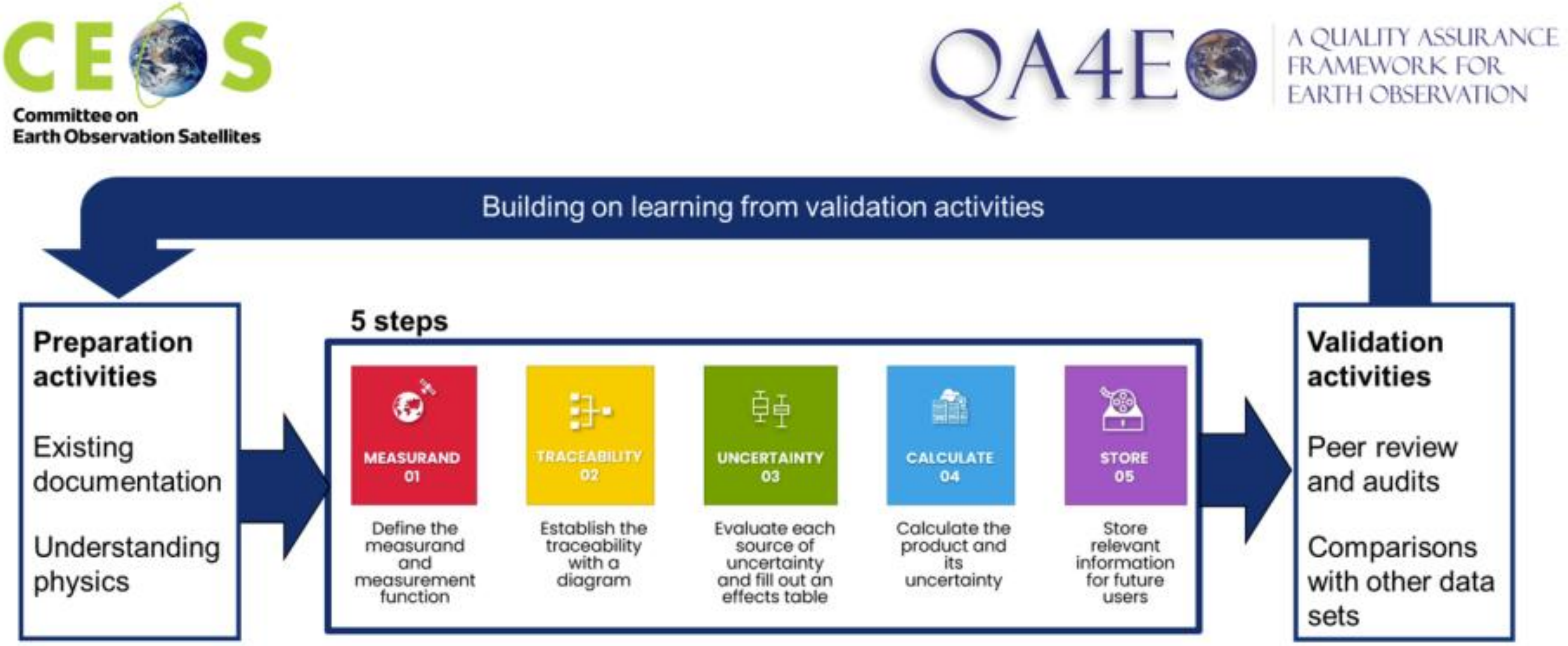
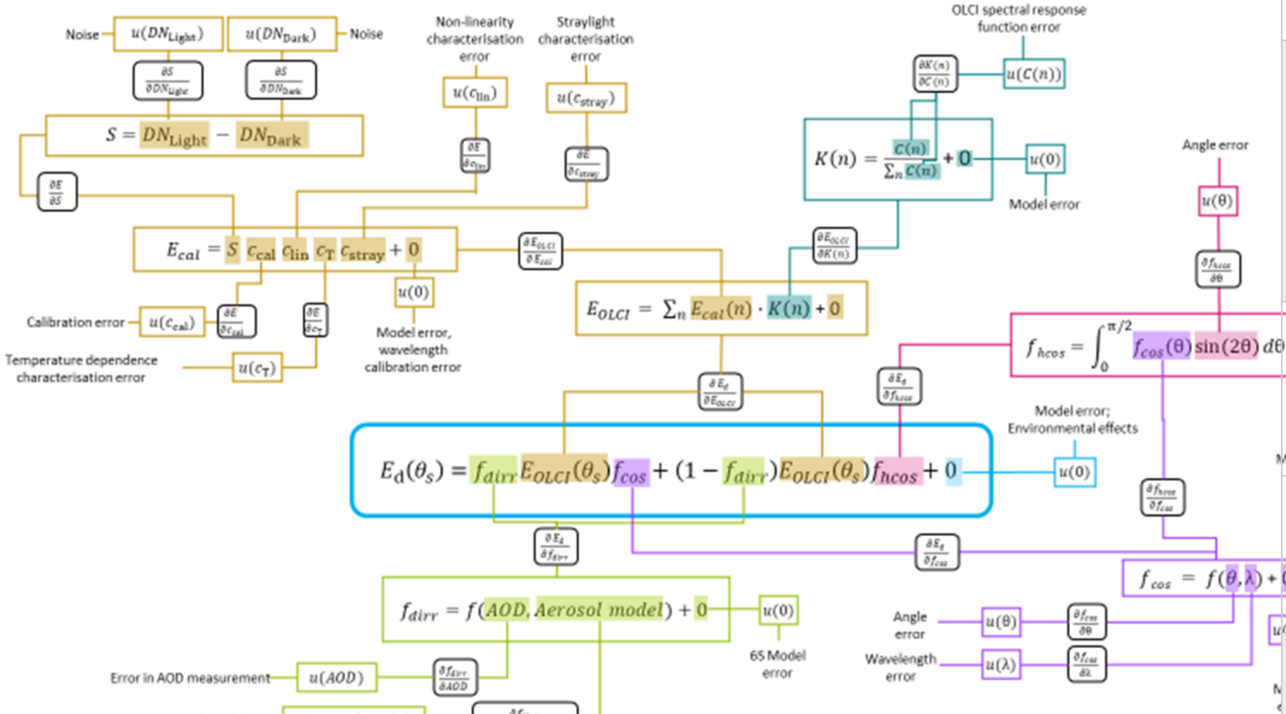


Figure 2 An iterative framework for the CEOS Five Steps

Methodology and resources

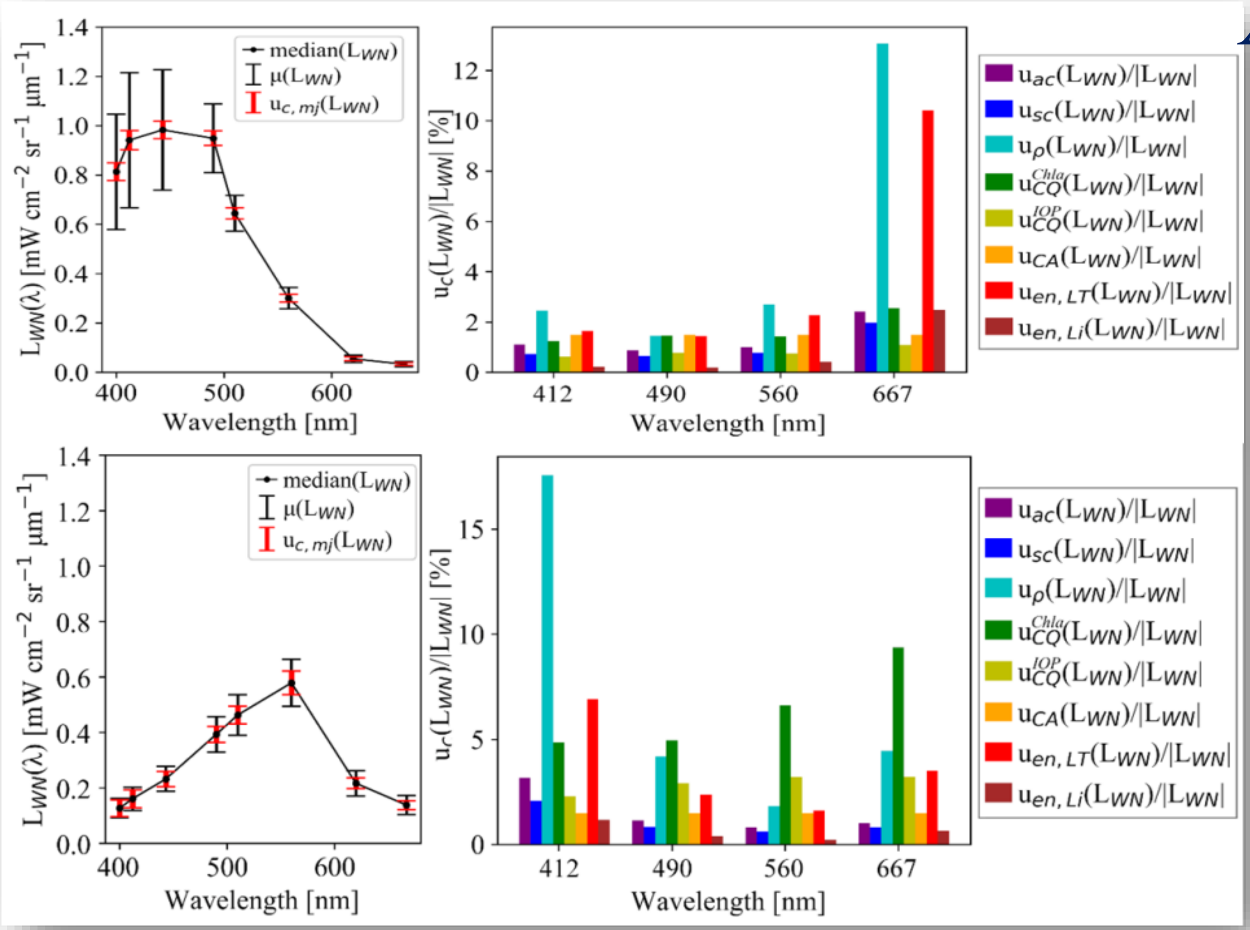
- FIDUCEO (FIDelity and Uncertainty in Climate data records from Earth Observations)



Downwelling Irradiance Uncertainty Tree Diagram

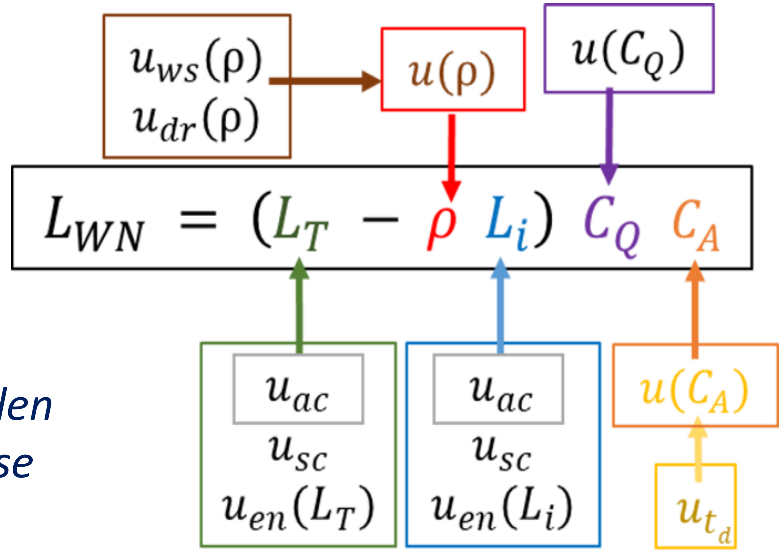
Table descriptor				
Name of effect		Detector calibration systematic error	Detector calibration random error 1, 2	Detector calibration stability model error 1, 2
Affected term in measurement function		c_{cal,z_1}, c_{cal,z_2}, $c_{cal,z_1,t}$, $c_{cal,z_2,t}$	c_{cal,z_1}, c_{cal,z_2}, $c_{cal,z_1,t}$, $c_{cal,z_2,t}$	c_{stab,z_1}, c_{stab,z_2}
Instruments in the series affected		All	All	All
Correlation type and form	Temporal within deployment	Rectangular Absolute	Rectangular Absolute	Rectangular Absolute
	Temporal between deployments	Rectangular Absolute	Random	Random
	Spectral (hyperspectral in-situ)	To be defined	To be defined	To be defined
Correlation scale	Temporal within deployment	-∞,+∞	-∞,+∞	-∞,+∞
	Temporal between deployments	a,b[1]	0	0
	Spectral (hyperspectral in-situ)	To be defined	To be defined	To be defined
Channels/bands	List of channels / bands affected	All	All	All
	Error correlation coefficient matrix	Identity – No correlation	Identity – No correlation	Identity – No correlation
Uncertainty	PDF shape	Gaussian	Gaussian	Gaussian
	units	Radiance/Counts	Radiance/Counts	Radiance/Counts
	magnitude	0.70%	0.25%	Less than 1%
Sensitivity coefficient		$\frac{\partial f}{\partial c_{cal\_s}}$	$\frac{\partial f}{\partial c_{cal\_r1}}$, $\frac{\partial f}{\partial c_{cal\_r2}}$	$\frac{\partial L_w}{\partial c_{stab1}}$, $\frac{\partial L_w}{\partial c_{stab2}}$,

AERONET-OC GUM application



Casablanca Platform

Gustaf Dalen Lighthouse



Relative uncertainty	400	412	443	490	510	560	620	667
$u_{c,mj}^{IOP}/L_{WN}$ (oligotrophic waters)	3.9	3.6	3.3	3.0	3.3	4.2	12.2	15.8
$u_{c,mj}^{IOP}/L_{WN}$ (optically complex waters)	22.3	18.7	11.1	5.9	5.1	4.5	5.8	6.7

Cazzaniga, I., & Zibordi, G. (2023). AERONET-OC L_{WN} Uncertainties: Revisited. *Journal of Atmospheric and Oceanic Technology*, *40*(4), 411-425.

Slide in courtesy of G. Zibor

CoMet

About Tools Examples News People

CoMet Toolkit

The **CoMet Toolkit** (Community Metrology Toolkit) is an open-source software project to develop Python tools for the handling of error-covariance information in the analysis of measurement data.

```
import xarray as xr
import obsarray
from punpy import MeasurementFunction, MCPropagation

# read digital effects table
ds = xr.open_dataset("digital_effects_table_gaslaw_example.nc")

# Define your measurement function inside a subclass of MeasurementFunction
class IdealGasLaw(MeasurementFunction):
    def meas_function(self, pres, temp, n):
        return (n *temp * 8.134)/pres

# Create Monte Carlo Propagation object, and create MeasurementFunction class
# object with required parameters such as names of input quantites in ds
prop = MCPropagation(10000)
gl = IdealGasLaw(prop, xvariables=["pressure", "temperature", "n_moles"],
                 yvariable="volume", yunit="m^3")

# propagate the uncertainties on the input quantites in ds to the measurand
# uncertainties in ds_y (propagate_ds returns random, systematic and structured)
ds_y = gl.propagate_ds(ds, store_unc_percent=True)
```

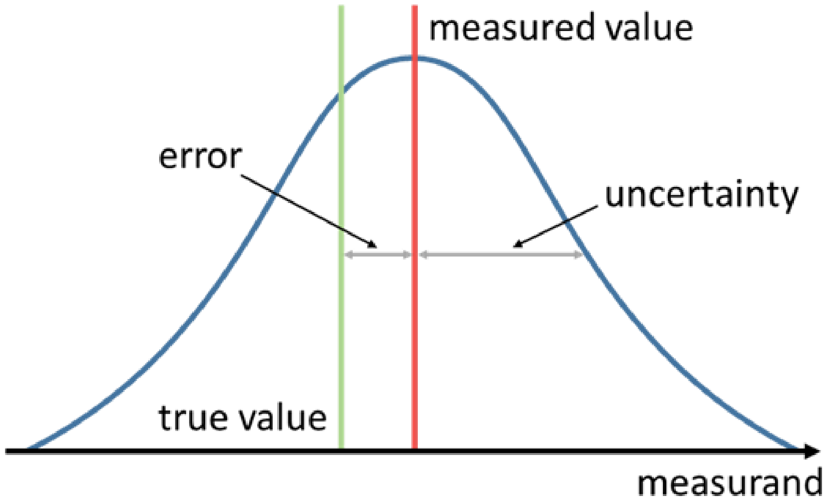
https://www.comet-toolkit.org/

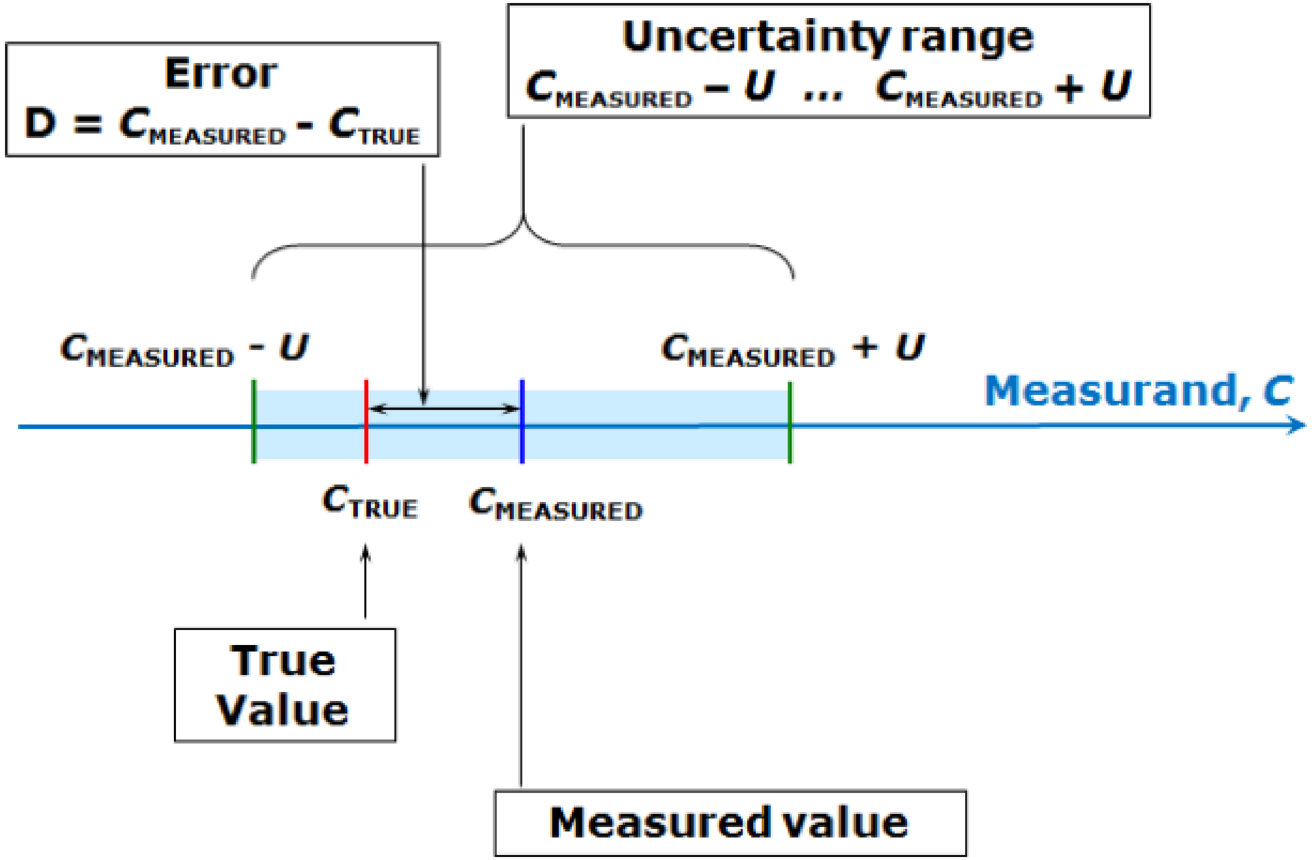
Basic uncertainty concepts

Error

is NOT the same as

Uncertainty





https://sisu-vana.ut.ee/measurement/introduction-concept-measurement-uncertainty

ESTIMATION OF MEASUREMENT UNCERTAINTY IN CHEMICAL ANALYSIS

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Search

2. THE ORIGIN OF MEASUREMENT UNCERTAINTY

Brief summary: Explanation, on the example of pipetting, where measurement uncertainty comes from. The concept of **uncertainty sources** – effects that cause the deviation of the measured value from the true value – is introduced. The main uncertainty sources of pipetting are introduced and explained: repeatability, calibration, temperature effect. Explanation of random and systematic effects is given. The concept of **repeatability** is introduced.

The first video demonstrates how pipetting with a classical volumetric pipette is done and explains where the uncertainty of the pipetted volume comes from.

https://sisu-vana.ut.ee/measurement/origin-measurement-uncertainty

Uncertainty vs. error

Uncertainty:

- Describes the spread of a probability distribution i.e. standard deviation

Error:

- The result of measurement imperfections
- From random and systematic effects

Correction

- Where an error is known, it can be corrected by applying a correction
- There will always be an unknown residual error

Consistency in terminology is important!

Systematic and random effects:

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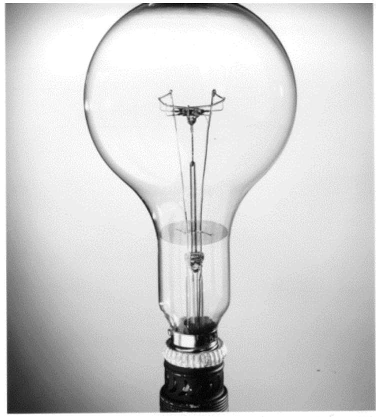
EFFECT	SYSTEMATIC
Calibration of reference	Yes
Alignment	If not realigned
Noise	No
Lamp current setting	Probably – if constant
Lamp current stability	Probably not
Temperature sensitivity	Depends on how much temperature is changing

lamp measured 5 times

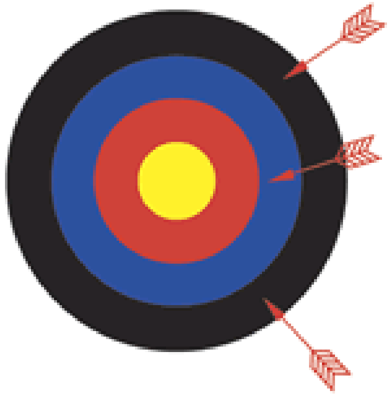


Effects are random or systematic depending on the measurement process itself

Measurement Uncertainty: Accuracy and Precision

Accuracy $\Rightarrow$ qualitative term relating the mean of the measurements to the true value

Precision $\Rightarrow$ represents the spread of the measurements



Poor precision, poor accuracy

Poor precision, good accuracy

Good precision, poor accuracy

Good precision, good accuracy

Situation	Random effects	Systematic effects	Uncertainty
1. 	Strong	Strong	High
2. 	Strong	Weak (or absent)	Medium
3. 	Weak	Strong	Medium
4. 	Weak	Weak (or absent)	Low

Scheme 2.1. The influence of random and systematic effects on measurement uncertainty.

https://sisu-vana.ut.ee/measurement/origin-measurement-uncertainty

What is not a measurement uncertainty?

- Mistakes made by operators are not measurement uncertainties. They should not be counted as contributing to uncertainty. They should be avoided by working carefully and by checking work.
- Accuracy (or rather inaccuracy) is not the same as uncertainty. Unfortunately, usage of these words is often confused. Correctly speaking, ‘accuracy’ is a qualitative term (e.g. you could say that a measurement was ‘accurate’ or ‘not accurate’). Uncertainty is quantitative. When a ‘plus or minus’ figure is quoted, it may be called an uncertainty, but not an accuracy.
- Errors are not the same as uncertainties (even though it has been common in the past to use the words interchangeably in phrases like ‘error analysis’).
- Statistical analysis is not the same as uncertainty analysis. Statistics can be used to draw all kinds of conclusions which do not by themselves tell us anything about uncertainty. Uncertainty analysis is only one of the uses of statistics.

Basic concepts

Uncertainty

Type A

Type B

Expanded

Standard

Coverage factor

Absolute

Relative

Effects of the errors

Systematic

Random

Correction

Uncertainty types

There are two methods for estimating uncertainties:

Type-A:

uncertainty estimates using statistics i.e. by taking multiple readings and using that information

Type-B:

uncertainty estimates from any other information, e.g. past experience, calibration certificates, etc.

Confidence intervals

- Uncertainty is given with respect to a given confidence interval:

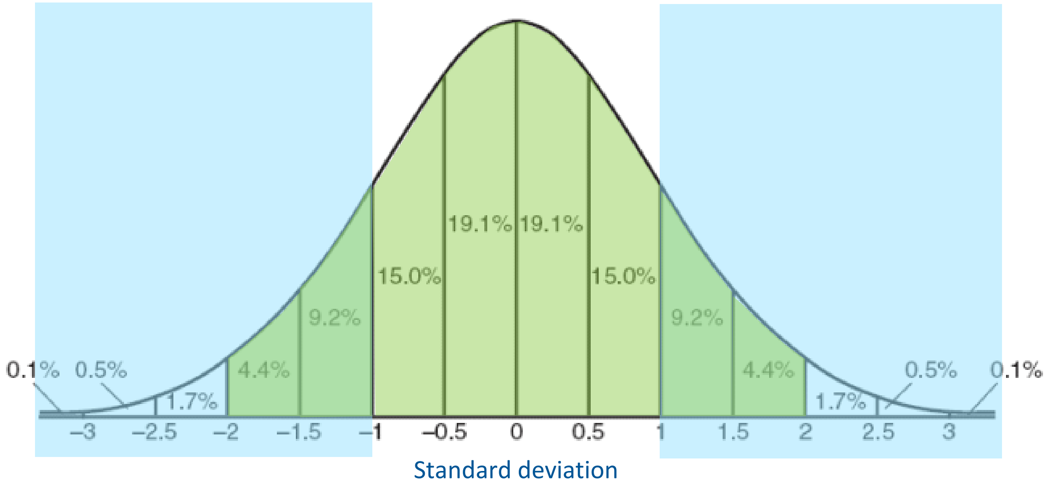
u(y) = 3 cm

at the 68.2% coverage probability (1σ or $k = 1$)

at the 95.4% confidence level

u(y) = 6 cm

at the 95.4% coverage probability (2σ or $k = 2$)



Uncertainty expression

Relative uncertainty:

5 mW m^{-2} nm^{-1} , $u = 0.2$ %

i.e. uncertainty expressed as a percentage

Absolute uncertainty:

5 mW m^{-2} nm^{-1} $u = 0.01$ mW m^{-2} nm^{-1} i.e. uncertainty expressed in the native measurement units

Standard deviation

- Describes the spread of the sample values about the mean
- A measure of the precision of the sample values
- The standard deviation is formalised as: $\sigma = \sqrt{\frac{(x_1-\bar{x})^2+(x_2-\bar{x})^2+(x_3-\bar{x})^2+(x_4-\bar{x})^2 \ldots+(x_n-\bar{x})^2}{n-1}} = \sqrt{\frac{\sum_{i=1}^{n}(x_i-\bar{x})^2}{n-1}}$

Sigma (lowercase) – used to denote the standard deviation

Standard uncertainty associated with the mean for small number of repeats

$$u^2_{\text{light,mean}} = \frac{N-1}{N-3}\left(\frac{s_{\text{light}}}{\sqrt{N}}\right)^2 .$$

N=5 , s*1.41
N=10, s*1.13
N=25, S*1,04

Standard uncertainty associated with the mean

- Tells us about the uncertainty associated with an average
- Expressed as *u(y)*: uncertainty associated with variable ‘y’
- Standard uncertainty is a margin whose size can be thought of as ± one standard deviation

$$u = \frac{\sigma}{\sqrt{n}}$$

Standard uncertainty associated with the mean

Standard deviation or spread of the results: Uncertainty associated with a single value

Number of samples

First order Taylor series approximation
uncorrelated input quantities version

$$u_c^2(y) = \sum_{i=1}^{n} \left(\frac{\partial f}{\partial x_i} \right)^2 u^2(x_i)$$

THE LAW OF PROPAGATION OF UNCERTAINTIES

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$$u_c^2(y) = \sum_{i=1}^{n} \left(\frac{\partial f}{\partial x_i} \right)^2 u^2(x_i)$$

Sensitivity Coefficients

Common derivatives

Common Derivatives

Polynomials

$$\frac{d}{dx}(c)=0 \qquad \frac{d}{dx}(x)=1 \qquad \frac{d}{dx}(cx)=c \qquad \frac{d}{dx}\left(x^n\right)=nx^{n-1} \qquad \frac{d}{dx}\left(cx^n\right)=ncx^{n-1}$$

From: http://tutorial.math.lamar.edu/

Simple case

$$V_S = V_{light} - V_{dark}$$

$$\frac{d}{dx}(x) = 1$$

$$\frac{\partial V_S}{\partial V_{light}} = 1 \qquad \frac{\partial V_S}{\partial V_{dark}} = -1$$

Very simple case
One minute of light radiometer readings
Followed by one minute of dark readings

Simple case

$$u_c^2(y)=\sum_{i=1}^{n}\left(\frac{\partial f}{\partial x_i}\right)^2 u^2(x_i)$$

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$$V_{\mathrm{S}}=V_{\mathrm{light}}-V_{\mathrm{dark}} \qquad \frac{\partial V_{\mathrm{S}}}{\partial V_{\mathrm{light}}}=1 \qquad \frac{\partial V_{\mathrm{S}}}{\partial V_{\mathrm{dark}}}=-1$$

$$u_c^2(V_{\mathrm{S}})=1^2u^2(V_{\mathrm{light}})+(-1)^2u^2(V_{\mathrm{dark}})$$

Absolute uncertainty

$$u(V_S)=\sqrt{u^2(V_{\mathrm{light}})+u^2(V_{\mathrm{dark}})}$$

Relative uncertainty

$$\frac{u(V_S)}{V_S}=\sqrt{\frac{u^2(V_{\mathrm{light}})+u^2(V_{\mathrm{dark}})}{(V_{\mathrm{light}}-V_{\mathrm{dark}})^2}}$$

Exercise 1

Calculate standard uncertainty of an instrument signal and report it as relative uncertainty.

Inputs in DN:

$$\overline{V}_{\text{light}} = 33031.00, s(V_{\text{light}}) = 2.19, N = 360$$

$$\overline{V}_{\text{dark}} = 32773.83, s(V_{\text{dark}}) = 2.16, N = 360$$

Where: $\overline{V}$ is arithmetic mean and s is standard deviation

Type A or Type B?

Exercise 1

Calculate standard uncertainty of an instrument signal and report it as relative uncertainty.

Type A

$$\frac{u(V_S)}{V_S} = \sqrt{\frac{u^2\left(\overline{V}_{\text{light}}\right) + u^2\left(\overline{V}_{\text{dark}}\right)}{\left(\overline{V}_{\text{light}} - \overline{V}_{\text{dark}}\right)^2}}$$

?

! Note to evaluate Type A uncertainty you need repeated observation

$$u^2(x) = s^2\left(\overline{x}\right) = \frac{s^2(x_i)}{n}$$

$s^2\left(\overline{x}\right)$ is experimental standard deviation of the mean

n is a number of repeated observation

Exercise 1

Calculate standard uncertainty of an instrument signal and report it as relative uncertainty.

Inputs in DN:

$$\overline{V}_{\text{light}} = 33031.00, s(V_{\text{light}}) = 2.19, N = 360$$

$$\overline{V}_{\text{dark}} = 32773.83, s(V_{\text{dark}}) = 2.16, N = 360$$

Where: $\overline{V}$ is arithmetic mean and s is experimental standard deviation

Results exercise 1

	Mean	Standard deviation	Number of readings	Absolute uncertainty (Type A) Standard deviation of the mean
Light [DN]	33031.000	2.190	360	0.115
Dark [DN]	32773.830	2.160	360	0.114

Measurements result and its combined standard uncertainty k = 1

Signal [DN]	Absolute uncertainty [DN] $u(V_S) = \sqrt{u^2(V_{\text{light}}) + u^2(V_{\text{dark}})}$	Relative uncertainty [%] $\frac{u(V_S)}{V_S}$
257.170 DN	0.162 DN	0.1 %

Exercise 1 concussions

- It might be easy to derive sensitivity coefficients from partial derivation

$$\frac{\partial V_{\mathrm{S}}}{\partial V_{\mathrm{light}}} = 1$$

- Once derived then a ready solution can be used in the future

$$V_{\mathrm{S}} = V_{\mathrm{light}} - V_{\mathrm{dark}} \qquad u(V_S) = \sqrt{u^2(V_{\mathrm{light}}) + u^2(V_{\mathrm{dark}})}$$

- Every measurement uncertainty can be expressed as absolute or relative, and converted from one to another. This is just more convenient to use one or the other for same type of calculations

- Type A uncertainty evaluation, is valid only for a very large number of repeated observation and uses standard deviation of the mean as a standard uncertainty

- THIS IS NOT THE CASE FOR TYBE B UNCERTAINTY EVALUATION

Sensitivity coefficients cheat sheets

Summation in quadrature for addition and subtraction

$e=a+b-c,$

$$\text{Combined uncertainty} = \sqrt{a^2 + b^2 + c^2 + \ldots etc}.$$

Summation in quadrature for multiplication or division

$A=L\cdot W,$

$$\frac{u(A)}{A} = \sqrt{\left(\frac{u(L)}{L}\right)^2 + \left(\frac{u(W)}{W}\right)^2}.$$

Sensitivity coefficients cheat sheets

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Squared value

Z^2,

$$\frac{2u(Z)}{Z}\ .$$

Summation in quadrature for more complicated function

$$P = \frac{V^2}{R},$$

$$\frac{u(P)}{P} = \sqrt{\left(\frac{2u(V)}{V}\right)^2 + \left(\frac{u(R)}{R}\right)^2}\ .$$

Absolute calibration measurement equation

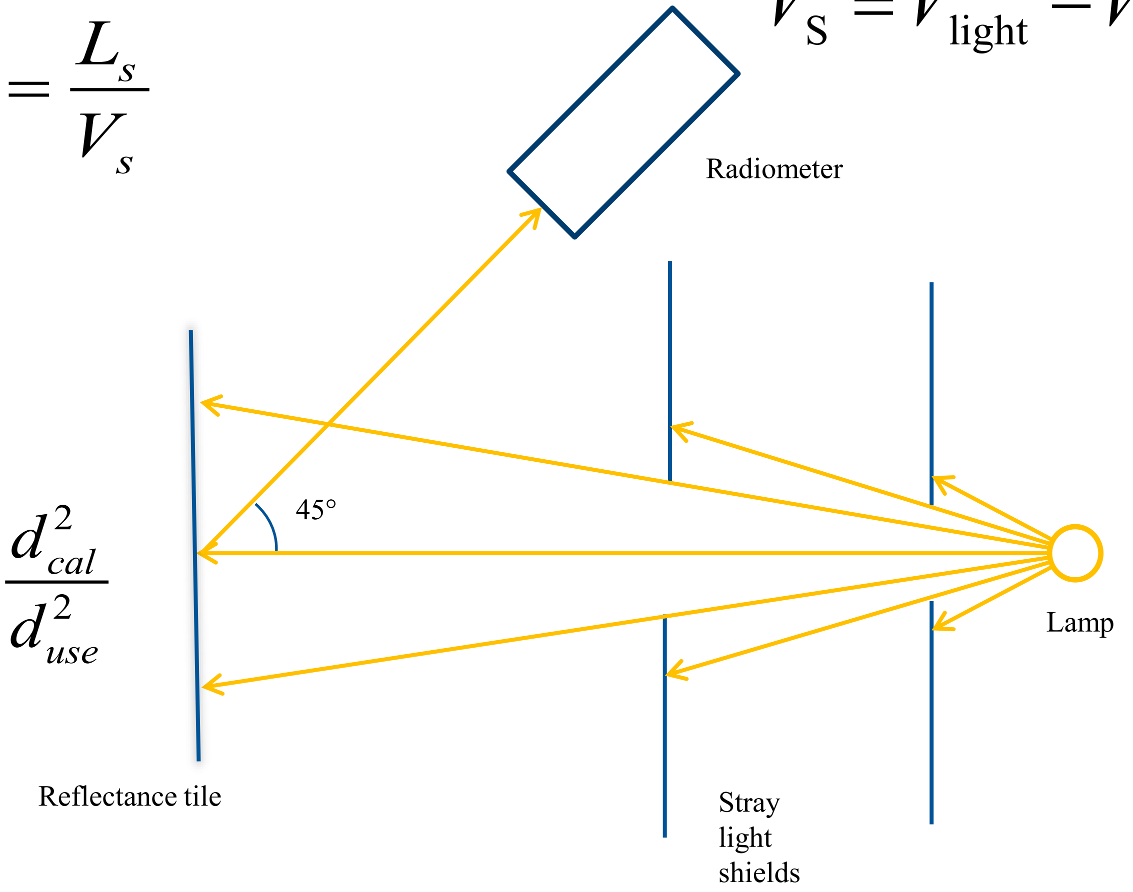
Calculation equation

Absolute calibration coefficient

$$A_L = \frac{L_s}{V_s}$$

$$V_S = V_{\text{light}} - V_{\text{dark}}$$

$$L_s = \frac{E_{FEL}\beta_{0:45}}{\pi}\frac{d_{cal}^2}{d_{use}^2}$$



Sources of uncertainty

Calibration certificate

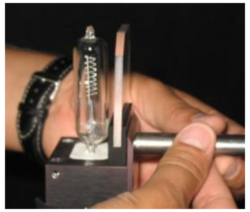
Lamp additional effects

- Ageing
- Alignment
- Current stability

Calibration certificate

Diffuser additional effects

- Ageing
- Uniformity



Distance accuracy

Random noise

Instrument additional effects

- Stability (drift)
- Room stray light

Measurement equations

$$L_s = \frac{E_{FEL}\beta_{0:45}}{\pi}\frac{d_{cal}^2}{d_{use}^2}$$

$$L_s = \frac{E_{FEL}\beta_{0:45}}{\pi}\frac{d_{cal}^2}{d_{use}^2} K_{lamp\_stab} K_{align} K_{current} K_{diff\_stab} K_{unif}$$

$$V_S = V_{light} - V_{dark}$$

$$V_S = V_{light} K_{light\_stab} + K_{stray} - V_{dark} K_{dark\_stab}$$

Calibration certificate

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Remember calibration certificates almost always quote uncertainties at $k = 2$!

Rectangular uncertainty distributions

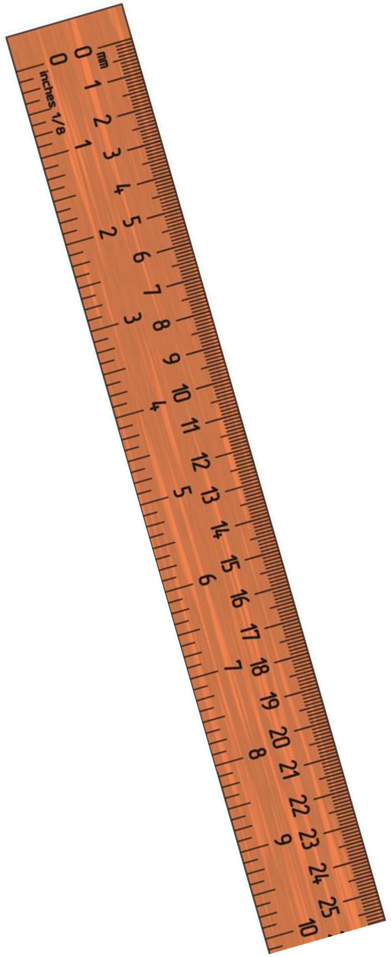
Resolution of distance measuring instrument = 0.1 mm

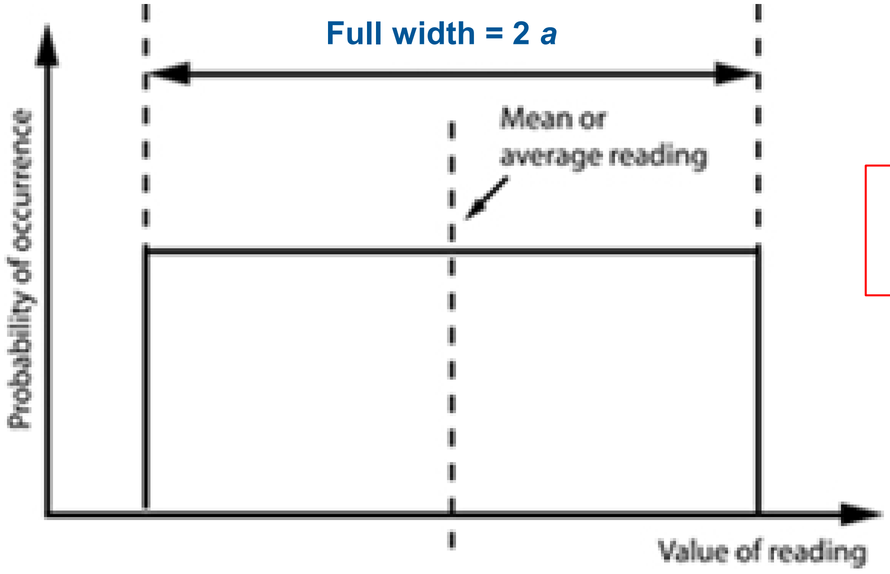
Measurement distance = 500.0 mm

Uncertainty associated with distance measurement = (0.05 / 500) / √3 = 0.006 %

Uncertainty in irradiance from distance measurement = 2 × 0.006 % = 0.012 %

$$-2 \cdot u\left(d_{\mathrm{use}}\right)/d_{\mathrm{use}}$$



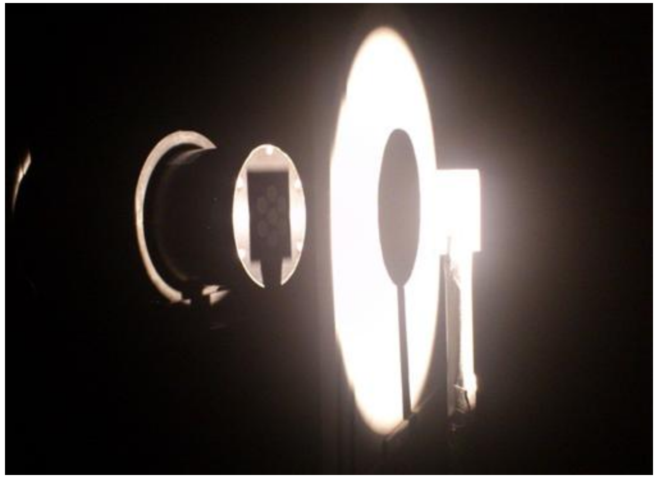


$$\frac{a}{\sqrt{3}}$$

Measurement equation: where to stop

- Room stray light negligible

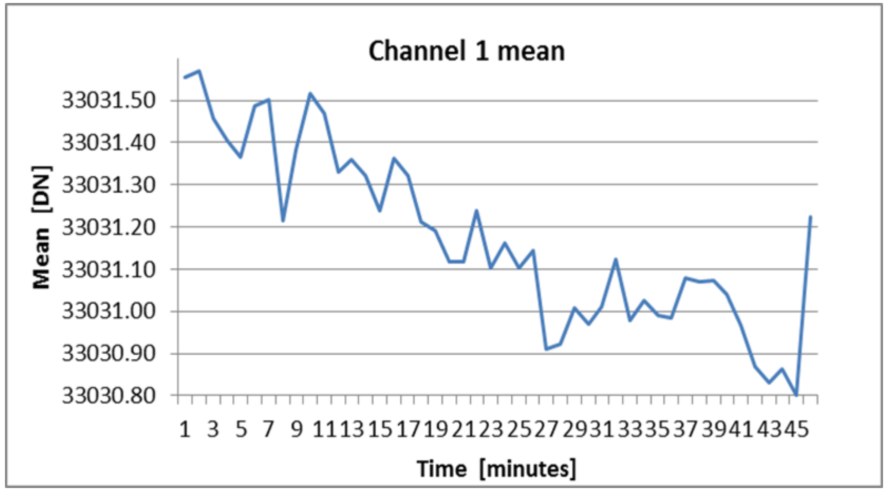
Difference between detector dark reading and measurement with detector FOV obscured smaller than standard deviation of individual dark runs



- No instrument drift

in controlled lab environment

0.7 DN change during
45 minute constant run



$$V_{\mathrm{S}} = V_{\mathrm{light}} \xcancel{K_{\mathrm{light\_stab}}} \xcancel{+ K_{\mathrm{stray}}} - V_{\mathrm{dark}} \xcancel{K_{\mathrm{dark\_stab}}}$$

Uncertainty budget

Symbol	Uncertainty component	Size of effect	Correction applied?	Residual uncertainty	Divisor	Sensitivity coefficient	Uncertainty associated with final value due to effect
$u(E_{\mathrm{FEL}})$	Ref. lamp irradiance	1.5 %	N	1.5 %	2	1	0.75 %
$u(\beta_{0:45})$	Tile radiance factor	2.0 %	N	2.0 %	2	1	1.00 %
$u(d_{\mathrm{use}})$	Lamp distance (500 mm)	0.05 mm	N	0.01 %	√3	2	0.012 %
$u(K_{\mathrm{align}})$	Lamp alignment	0.15 %	N	0.15 %	1	1	0.15 %
$u(K_{\mathrm{l\_stab}})$	Light reading stability	negligible	N	negligible			negligible
$u(K_{\mathrm{d\_stab}})$	Dark reading stability	negligible	N	negligible			negligible
$u(K_{\mathrm{lamp\_stab}})$	Lamp stability	0.083 %	N	0.083 %	√3	1	0.048 %
$u(K_{\mathrm{diff\_stab}})$	Diffuser stability	0.125 %	N	0.125 %	√3	1	0.072 %
$u(K_{\mathrm{stray}})$	Stray light in lab	negligible	N	negligible			negligible
$u(K_{\mathrm{current}})$	Lamp current (8.000 A)	0.004 A	N	0.25 % in *I*, or 0.99 % in E_{FEL} at 600 nm	√3	1	0.572 % (at 600 nm)
$u(K_{\mathrm{unif}})$	Radiance uniformity	1.50 %	N	1.50 %	√3	1	0.866 %
Combined standard uncertainty							1.63 %
Expanded uncertainty (*k*=2)							3.3 %

Uncertainty budget

1. Traceability Chain

Uncertainty evaluation type ?

Type B – information form calibration certificates !

Symbol	Uncertainty component	Size of effect	Correction applied?	Residual uncertainty	Divisor	Sensitivity coefficient	Uncertainty associated with final value due to effect
$u(E_{\mathrm{FEL}})$	Ref. lamp irradiance	1.5 %	N	1.5 %	2	1	0.75 %
$u(\beta_{0:45})$	Tile radiance factor	2.0 %	N	2.0 %	2	1	1.00 %

Coverage factor ?
Probability distribution?

k = 2, Gaussian

Uncertainty budget

2. Calculation equation

Uncertainty evaluation type ?

Type B – information form calibration certificates
Type A – repeated measurements

Symbol	Uncertainty component	Size of effect	Correction applied?	Residual uncertainty	Divisor	Sensitivity coefficient	Uncertainty associated with final value due to effect
$u(E_{\mathrm{FEL}})$	Ref. lamp irradiance	1.5 %	N	1.5 %	2	1	0.75 %
$u(\beta_{0:45})$	Tile radiance factor	2.0 %	N	2.0 %	2	1	1.00 %
$u(d_{\mathrm{use}})$	Lamp distance (500 mm)	0.05 mm	N	0.01 %	√3	2	0.012 %

Absolute uncertainty

Relative uncertainty

Uncertainty budget

3. Sources of uncertainty
4. Measurement equation (all components with assigned size of effect)
5. Sensitivity coefficient

Symbol	Uncertainty component	Size of effect	Correction applied?	Residual uncertainty	Divisor	Sensitivity coefficient	Uncertainty associated with final value due to effect
$u(E_{FEL})$	Ref. lamp irradiance	1.5 %	N	1.5 %	2	1	0.75 %
$u(\beta_{0:45})$	Tile radiance factor	2.0 %	N	2.0 %	2	1	1.00 %
$u(d_{use})$	Lamp distance (500 mm)	0.05 mm	N	0.01 %	√3	2	0.012 %
$u(K_{align})$	Lamp alignment	0.15 %	N	0.15 %	1	1	0.15 %
$u(K_{l\_stab})$	Light reading stability	negligible	N	negligible			negligible
$u(K_{d\_stab})$	Dark reading stability	negligible	N	negligible			negligible
$u(K_{lamp\_stab})$	Lamp stability	0.083 %	N	0.083 %	√3	1	0.048 %
$u(K_{diff\_stab})$	Diffuser stability	0.125 %	N	0.125 %	√3	1	0.072 %
$u(K_{stray})$	Stray light in lab	negligible	N	negligible			negligible
$u(K_{current})$	Lamp current (8.000 A)	0.004 A	N	0.25 % in *I*, or 0.99 % in E_{FEL} at 600 nm	√3	1	0.572 % (at 600 nm)
$u(K_{unif})$	Radiance uniformity	1.50 %	N	1.50 %	√3	1	0.866 %
Combined standard uncertainty							1.63 %
Expanded uncertainty (*k*=2)							3.3 %

Uncertainty budget

6. Assigning Uncertainties
7. Combining your uncertainties

Symbol	Uncertainty component	Size of effect	Correction applied?	Residual uncertainty	Divisor	Sensitivity coefficient	Uncertainty associated with final value due to effect
$u(E_{FEL})$	Ref. lamp irradiance	1.5 %	N	1.5 %	2	1	0.75 %
$u(\beta_{0:45})$	Tile radiance factor	2.0 %	N	2.0 %	2	1	1.00 %
$u(d_{use})$	Lamp distance (500 mm)	0.05 mm	N	0.01 %	√3	2	0.012 %
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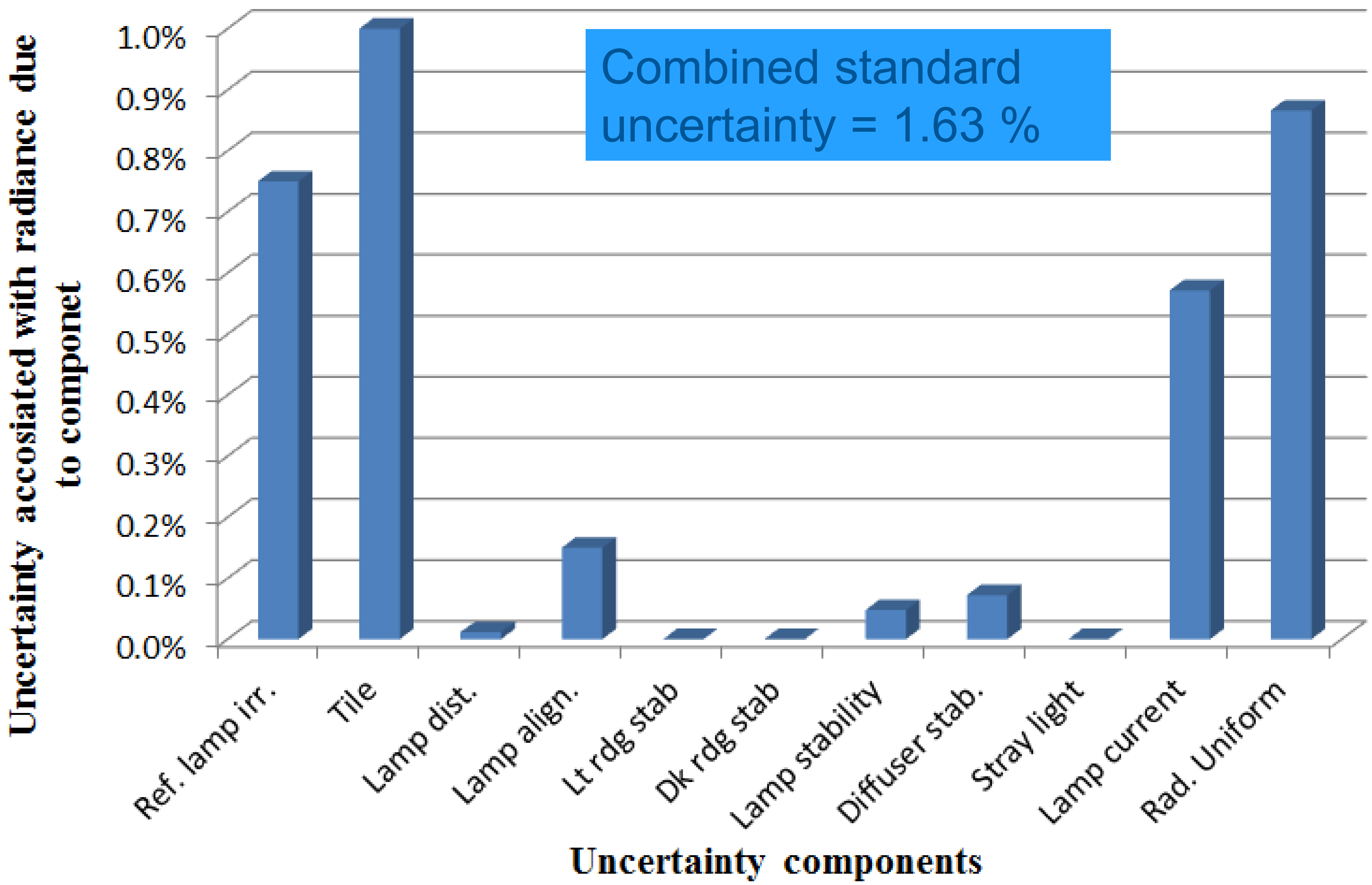
$u(K_{unif})$	Radiance uniformity	1.50 %	N	1.50 %	√3	1	0.866 %
Combined standard uncertainty							1.63 %
Expanded uncertainty (*k*=2)							3.3 %

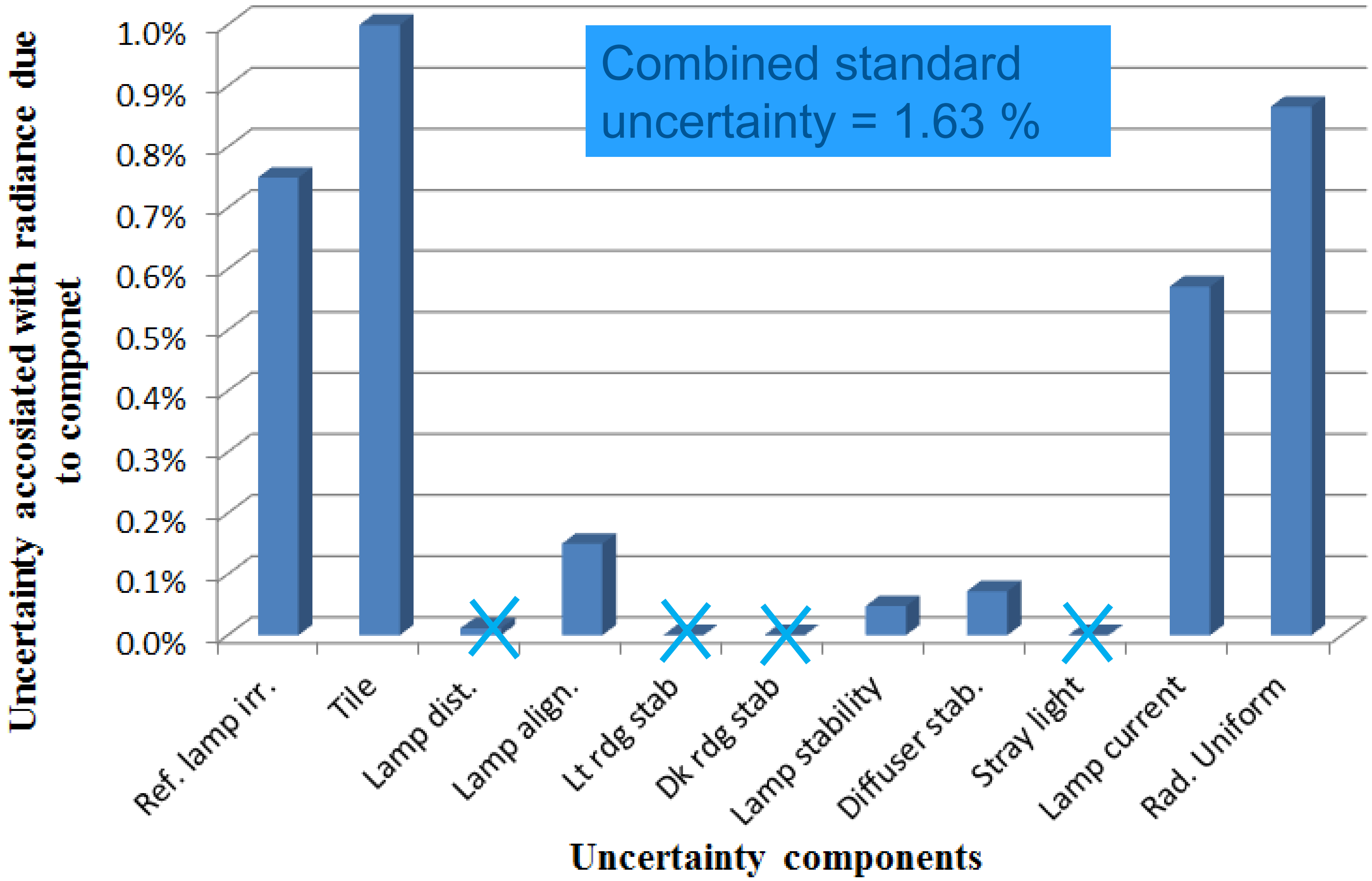
8. Expanding your uncertainties

When to stop



Uncertainty accosiated with radiance due to componet
1.0%
0.9%
0.8%
0.7%
0.6%
0.5%
0.4%
0.3%
0.2%
0.1%
0.0%
Combined standard uncertainty = 1.63 %
Ref. lamp irr.
Tile
Lamp dist.
Lamp align.
Lt rdg stab
Dk rdg stab
Lamp stability
Diffuser stab.
Stray light
Lamp current
Rad. Uniform
Uncertainty components

When to stop



Combined standard uncertainty = 1.63 %
Uncertainty accosiated with radiance due to componet
1.0%
0.9%
0.8%
0.7%
0.6%
0.5%
0.4%
0.3%
0.2%
0.1%
0.0%
Ref. lamp irr.
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Lamp align.
Lt rdg stab
Dk rdg stab
Lamp stability
Diffuser stab.
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Lamp current
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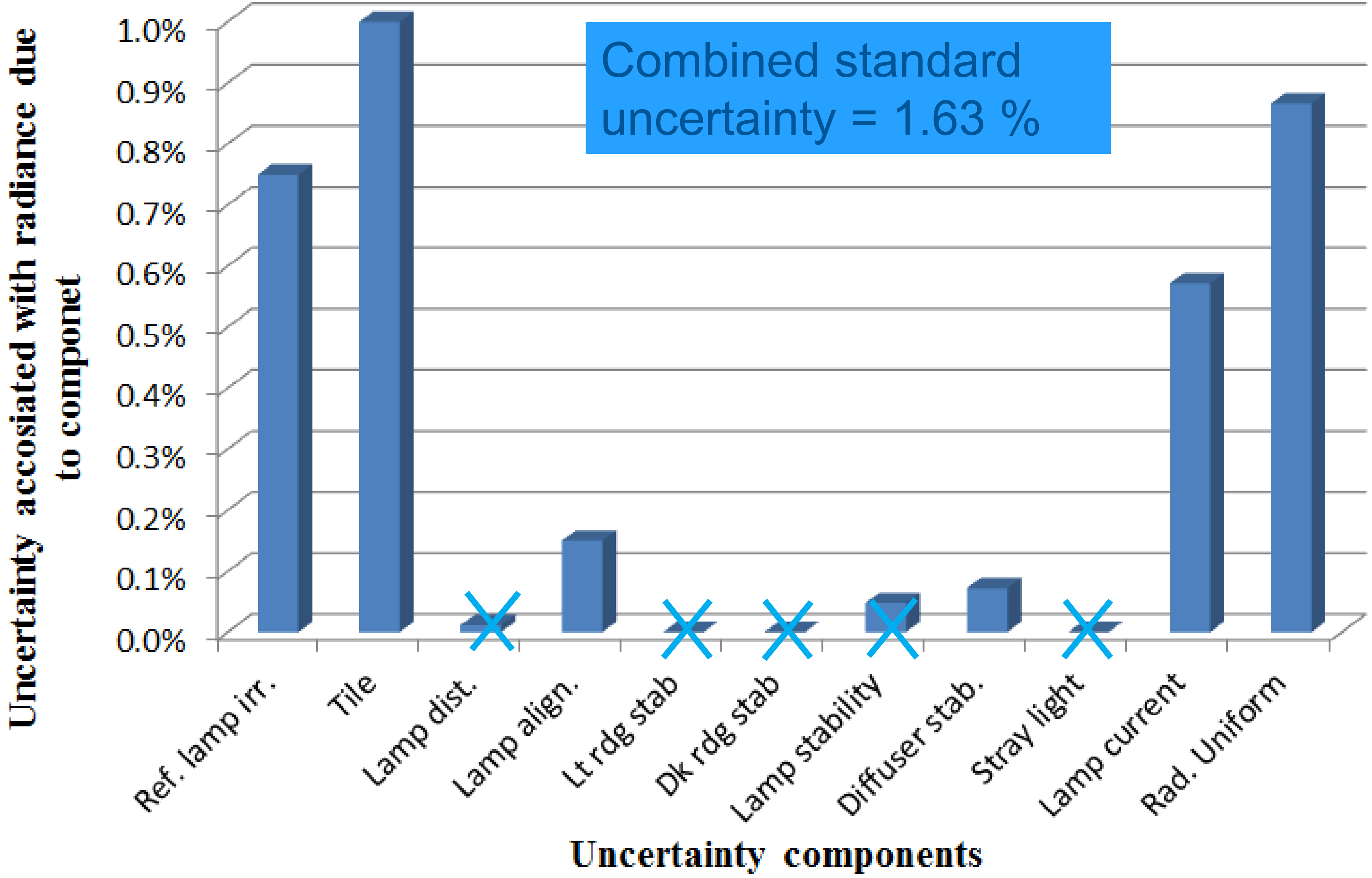
Uncertainty components

When to stop



Combined standard uncertainty = 1.63 %
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1.0%
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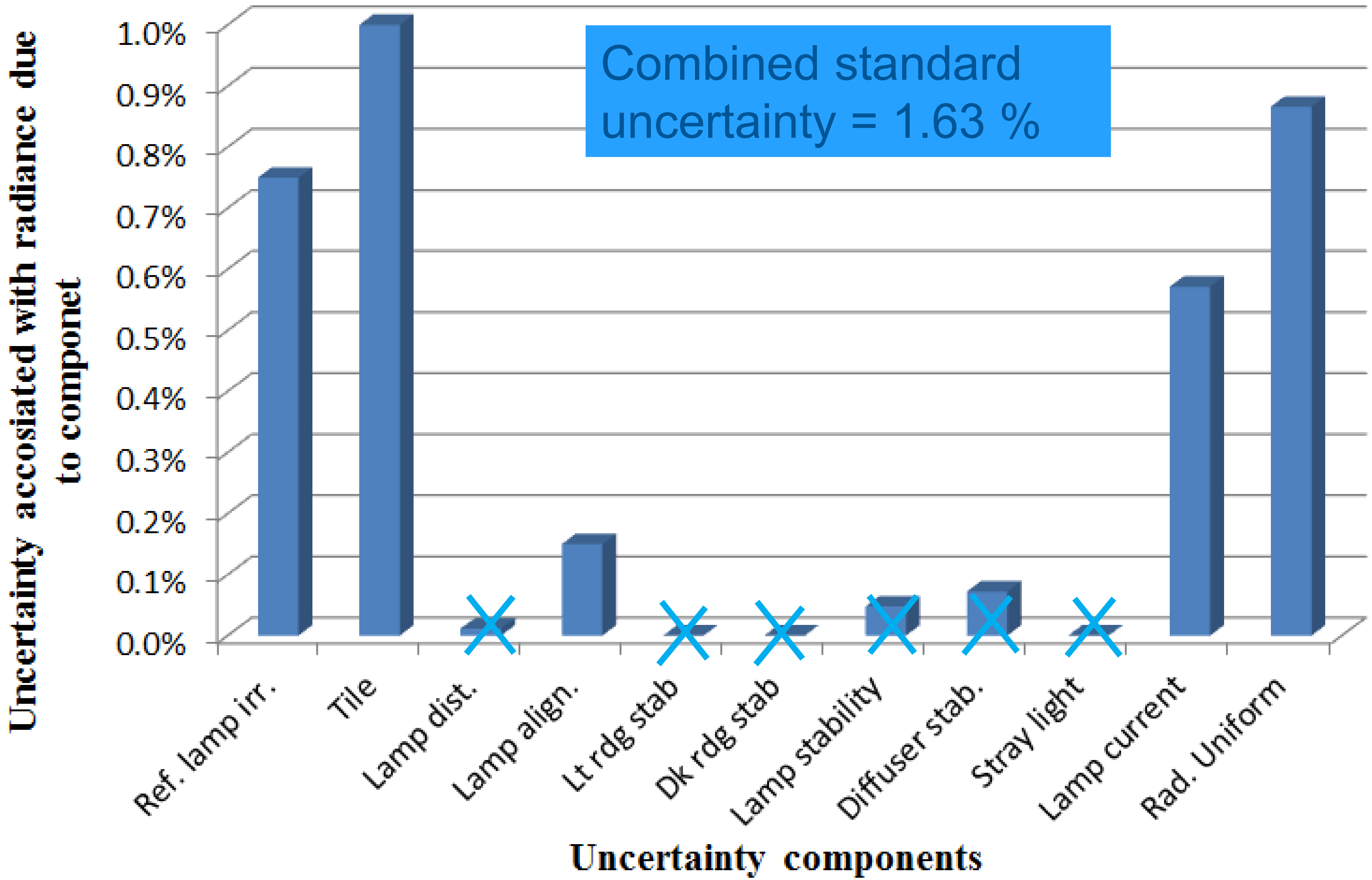
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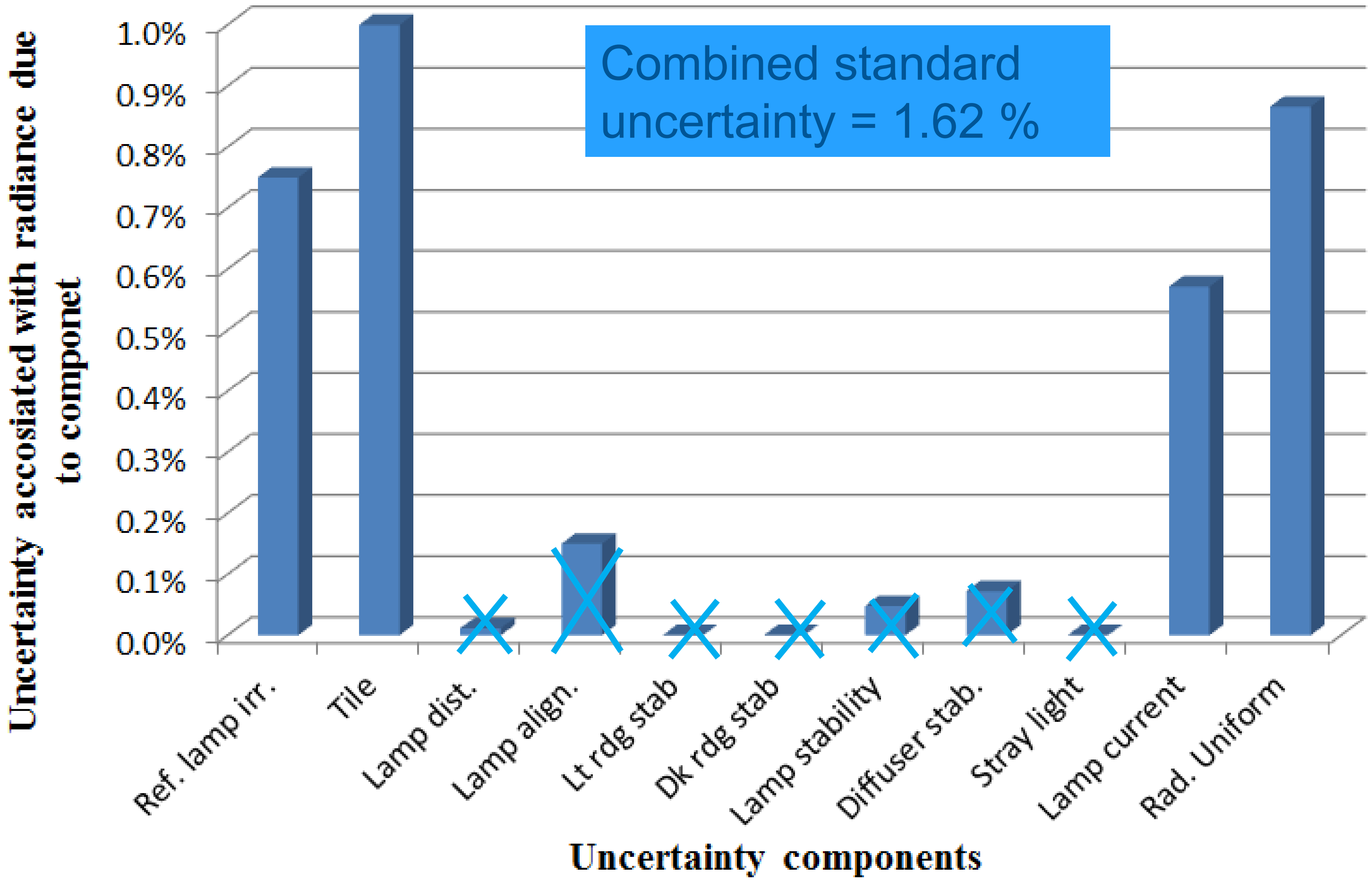
When to stop



Combined standard uncertainty = 1.62 %
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Ref. lamp irr.
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Lamp align.
Lt rdg stab
Dk rdg stab
Lamp stability
Diffuser stab.
Stray light
Lamp current
Rad. Uniform
Uncertainty components

Above water radiometry measurement equation

LPU example for water leaving radiance

$$u_c^2(y) = \sum_{i=1}^{n} \left(\frac{\partial f}{\partial x_i} \right)^2 u^2(x_i)$$

$$y = f(x_1, x_2, \ldots, x_N)$$

$$L_w = L_t - \rho * L_i$$

We assume uncertainty in all inputs components are due to random effects, thus there is no error correlation.
We assume that uncertainty in all inputs components have Normal distribution.

Sensitivity coefficients

$$L_w = L_t - \rho * L_i$$

$$\frac{\partial L_w}{\partial L_t} = 1,$$

$$\frac{\partial L_w}{\partial \rho} = L_i,$$

$$\frac{\partial L_w}{\partial L_i} = \rho,$$

Combined uncertainty

$$u(L_w) = \sqrt{1^2 \cdot u(L_t)^2 + L_i^2 \cdot u(\rho)^2 + \rho^2 \cdot u(L_i)^2}$$

All uncertainties here are expresses as an absolute uncertainty

LPU example for Remote sensing reflectance

$$\frac{\partial R_{rs}}{\partial L_w} = \frac{1}{E_s},$$

$$\frac{\partial R_{rs}}{\partial E_s} = -\frac{L_w}{E_s^2},$$

$$R_{rs} = \frac{L_w}{E_s},$$

$$u(R_{rs}) = \sqrt{\left(\frac{1}{E_s}\right)^2 \cdot u(L_w)^2 + \left(-\frac{L_w}{E_s^2}\right)^2 \cdot u(E_s)^2}$$

All uncertainties here are expresses as an absolute uncertainty

Remote sensing reflectance as relative uncertainty

$$u(R_{rs}) = \sqrt{\left(\frac{1}{E_s}\right)^2 \cdot u(L_w)^2 + \left(-\frac{L_w}{E_s^2}\right)^2 \cdot u(E_s)^2}$$

Divide both sites of equation by R_{rs}

Relative uncertainty in remote sensing reflectance

$$\frac{u(R_{rs})}{R_{rs}} = \sqrt{\frac{\left(\frac{1}{E_s}\right)^2 \cdot u(L_w)^2}{R_{rs}^2} + \frac{\left(-\frac{L_w}{E_s^2}\right)^2 \cdot u(E_s)^2}{R_{rs}^2}}$$

Relative uncertainty in remote sensing reflectance

$$\frac{u(R_{rs})}{R_{rs}} = \sqrt{\frac{\left(\frac{1}{E_s}\right)^2 \cdot u(L_w)^2}{\left(L_w/E_s\right)^2} + \frac{\left(-\frac{L_w}{E_s^2}\right)^2 \cdot u(E_s)^2}{\left(L_w/E_s\right)^2}}$$

Relative uncertainty in remote sensing reflectance

$$\frac{u(R_{rs})}{R_{rs}} = \sqrt{\frac{u(L_w)^2}{(L_w)^2} + \frac{u(E_s)^2}{(E_s)^2}}$$

Remote sensing reflectance cheat sheet

$$R_{rs} = \frac{L_t - \rho L_i}{E_s}$$

$$u_{rel}(L_r) = \sqrt{u_{rel}(\rho)^2 + u_{rel}(L_i)^2}$$

$$u_{asb}(L_w) = \sqrt{u_{abs}(L_t)^2 + u_{abs}(L_r)^2}$$

$$u_{rel}(L_w) = \sqrt{\frac{u_{abs}(L_t)^2 + u_{abs}(L_r)^2}{L_w^2}}$$

$$u_{rel}(R_{rs}) = \sqrt{u_{rel}(L_w)^2 + u_{rel}(E_s)^2}$$

Take home message

So called sum of squares for relative (%) uncertainty estimation is correct only for equations with multiplications and divisions as mathematical operations.

It works for additions and substations as well but then uncertainties as absolute, thus in the same units and input components.

It doesn’t work directly for equation where both multiplication and addition occurs such as L_w.

Congratulation!

I finished and you survived ;-)